TAFTCOLLEGE

Math 2100 Analytic Geometry and Calculus I

Catalog Description:

This course is a beginning course in calculus and analytic geometry including functions, limits and continuity, derivatives, integrals, applications of derivatives and integrals, transcendental functions, and Fundamental Theorem of Calculus. This course is primarily for Science, Technology, Engineering, and Math majors and is taught with a computer component (Maple). C-ID: MATH 211. Transfer Credit: CSU; UC.

SLO:

Course #1 - Calculate limits
Course #2 - Calculate and interpret instantaneous rates of change.
Course #3 - Calculate the area under a curve.

Sample Problems:

1. Graph each of the following functions. Make sure to state the domain and range of each function as well.
   a. $f(x) = 5 + \sqrt{x+3}$

   b. $g(t) = -|t-5| + 2$

   c. $h(x) = 2x^2 - 2x - 24$

   d. $f(\theta) = 3sin(2\theta)$

   e. $g(t) = \frac{3}{t-1}$

   f. $h(x) = 2^{x-1} + 5$

   g. $p(x) = \begin{cases} x^2 - 1, & x < -2 \\ -3x - 3, & -2 \le x \le 5 \\ -18, & x > 5 \end{cases}$

2. Let $f(x) = x^3 + 2, g(x) = \frac{1}{x}$, & $h(x) = sin(x)$. Use these functions to evaluate the following.

a. $f(5)$

b. $g(h(x))$

c. $f(x) \cdot h(x)$

d. $h\left(\frac{\pi}{2}\right) \cdot g(2)$

e. $f\left(g(h(x))\right)$

f. $h(f(x))$

g. $g\left(\frac{1}{2}\right)$

h. $f(x)^2 \cdot g(x)$

i. $g(f(3))$

3. Solve the following equations for y.

a. $2xy + x^2 + sin(x) = 3y - 4xy + 2$

b. $sin(y) + 5 = x^2 + 3x + 2$

c. $\log_{17} y = x + 2$

d. $\frac{1}{y+2} + x^2 = cos(x)$

e. $(x + 2y)^2 - 3x + sin(0) = 4y^2 + 2xy$

4. Evaluate the following value if it exists.

a. $\sin\left(\frac{5\pi}{2}\right)$

b. $\cos\left(-\frac{\pi}{3}\right)$

c. $\sec(0)$

d. $\cos\left(\frac{7\pi}{3}\right)$

e. $\tan\left(-\frac{\pi}{4}\right)$

f. $\cot\left(\frac{13\pi}{6}\right)$

g. $\csc(2\pi)$

5. Simplify the following expressions

a. $(a^2)^3 + (3 + b)^2$

b. $(x^4 + 7x^4 - 5x^2)^2$

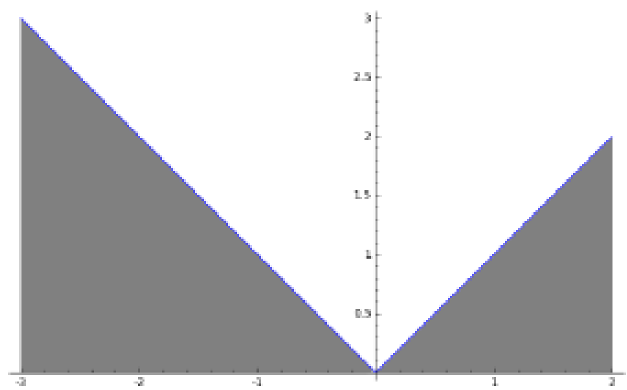
c. $4(5 - 3p) - (2 + 4p) - 3(2 + 10p)$

d. $\sin(0) + \ln(e) + \cos(\pi)$

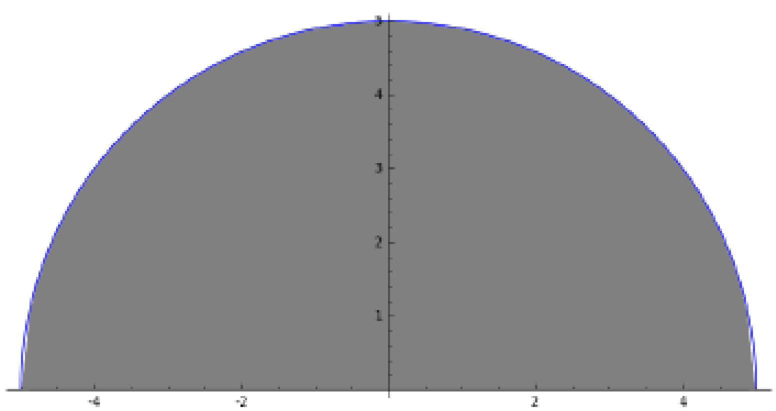
e. $\frac{t^{-3}+s^2t^2+5}{t^5}$

6. Find the area of the shaded regions below.

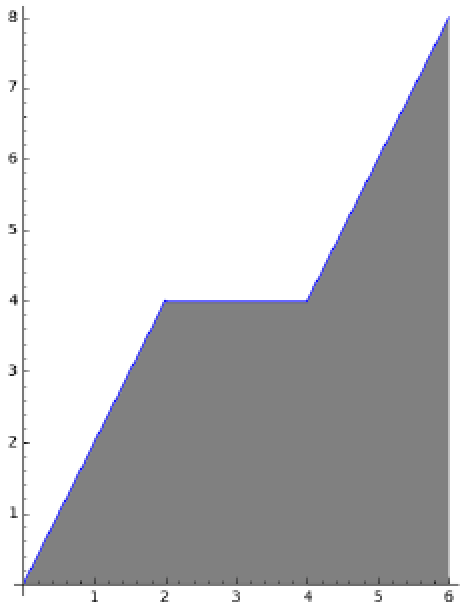
a. Area between $f(x) = |x|$ and the $x-$axis from $x = -3$ to $x = 2$.



b. Area between $f(x) = \sqrt{25 - x^2}$ and the $x-$axis from $x = -5$ to $x = 5$.



c. Area between the piecewise function below and the $x - axis$ from $x = 0$ to $x = 6$.



7. Find all vertical and horizontal asymptotes for the following functions

a. $\frac{1}{(x-3)^2}$

b. $\frac{2x-5}{(5x-3)}$

c. $\frac{(x-3)(x-2)(x+5)}{x+4}$

d. $\frac{(2x+3)(4x-7)}{x^2-4}$

8. Find a linear function $f(x)$ such that $f(3) = 7$ and $f(-1) = \frac{3}{2}$.

9. Find the $x -$ intercepts for the following functions.

a. $f(x) = (x-3)^2(x^2 + 4x + 3)$

b. $g(\theta) = \sin(\theta)$

c. $h(x) = \frac{5}{x-3} + 1$

10. Find the distance between the points $P(2,5)$ and $Q(3,-7)$.

11. Rewrite each of the following radical expressions in exponential form, and then simplify the result as much as possible.

a. $\sqrt[3]{x}$

b. $\sqrt[3]{x^{12}}$

c. $\left(\sqrt[2]{x}\right)^5$

d. $\sqrt[2]{x^5}$

e. $\frac{1}{\sqrt[3]{x^2}}$

f. $\frac{\sqrt[2]{x}}{\sqrt[3]{x}}$

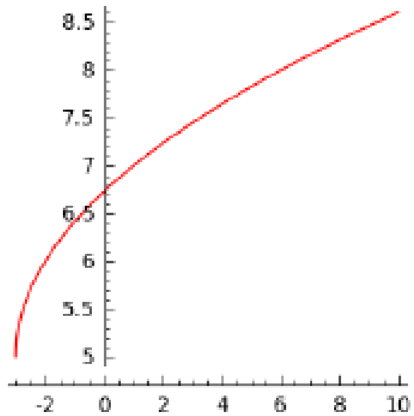
g. $\sqrt[3]{x}\left(\frac{1}{\sqrt{x}} + x - 2 + \frac{1}{\sqrt[3]{x}}\right)$

h. $\frac{\sqrt[7]{x^2}+\sqrt[3]{x^3}}{\sqrt{x}}$

Solutions:

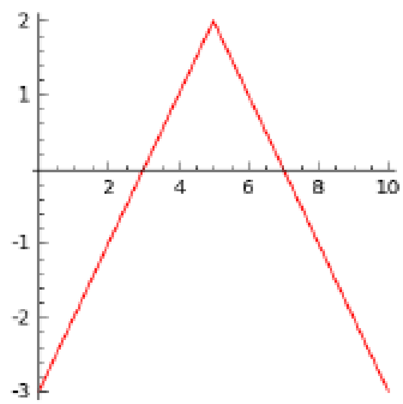
1. Graph each of the following functions. Make sure to state the domain and range of each function as well.

   a. $f(x) = 5 + \sqrt{x+3}$



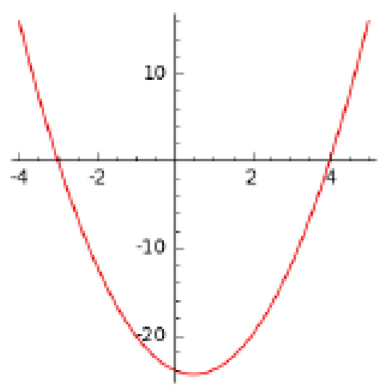
Domain: $[-3, \infty)$
Range: $[5, \infty)$

   b. $g(t) = -|t-5| + 2$



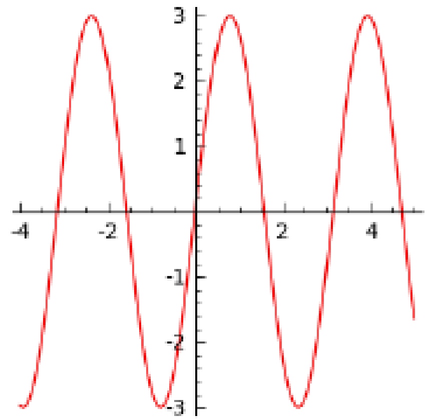
Domain: $(-\infty, \infty)$
Range: $(-\infty, 2]$

   c. $h(x) = 2x^2 - 2x - 24$



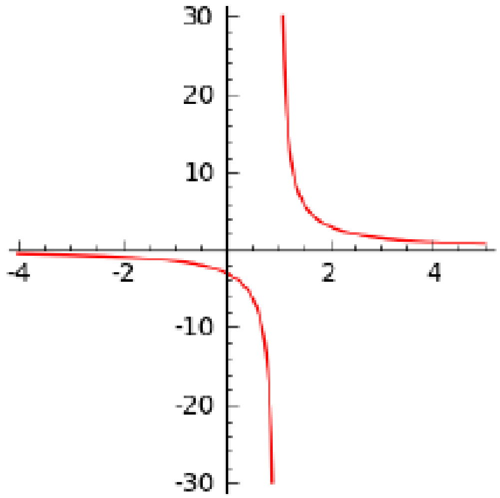
Domain: $(-\infty, \infty)$
Range: $[-24.5, \infty)$

d. $f(\theta) = 3sin(2\theta)$



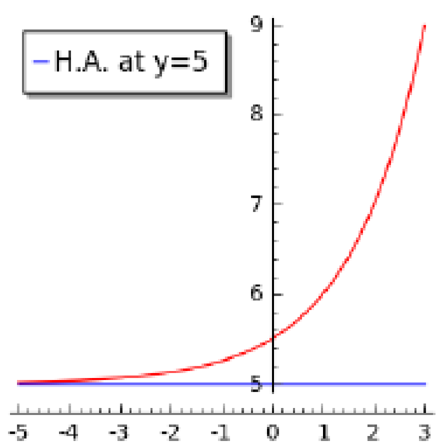
Domain: $(-\infty, \infty)$
Range: $[-3,3]$

e. $g(t) = \frac{3}{t-1}$



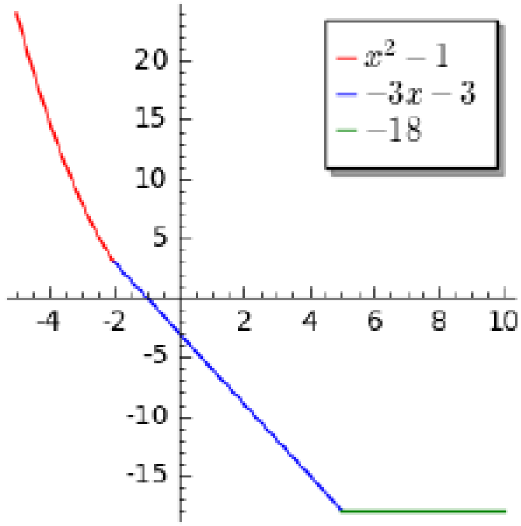
Domain: $(-\infty, 1) \cup (1, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$

f. $h(x) = 2^{x-1} + 5$



Domain: $(-\infty, \infty)$
Range: $(5, \infty)$

g. $p(x) = \begin{cases} x^2 - 1, & x < -2 \\ -3x - 3, & -2 \leq x \leq 5 \\ -18, & x > 5 \end{cases}$



Domain: $(-\infty, \infty)$
Range: $[-18, \infty)$

2. Let $f(x) = x^3 + 2, g(x) = \frac{1}{x},$ & $h(x) = sin(x)$. Use these functions to evaluate the following.

a. $f(5) = 5^3 + 2 = 127$

b. $g(h(x)) = g(\sin(x)) = \frac{1}{\sin(x)} = \csc(x)$

c. $f(x) \cdot h(x) = (x^3 + 2) \cdot (\sin(x)) = x^3 \sin(x) + 2\sin(x)$

d. $h\left(\frac{\pi}{2}\right) \cdot g(2) = \sin\left(\frac{\pi}{2}\right) \cdot \left(\frac{1}{2}\right) = 1 \cdot \frac{1}{2} = \frac{1}{2}$

e. $f\left(g(h(x))\right) = f\left(\frac{1}{\sin(x)}\right) = f(\csc(x)) = \csc^3(x) + 2$

f. $h(f(x)) = \sin(x^3 + 2)$

g. $g\left(\frac{1}{2}\right) = \frac{1}{\frac{1}{2}} = 2$

h. $f(x)^2 \cdot g(x) = (x^3 + 2)^2 \cdot \left(\frac{1}{x}\right) = (x^6 + 4x^3 + 4) \cdot \left(\frac{1}{x}\right) = x^5 + 4x^2 + \frac{4}{x}$

i. $g(f(3)) = g(3^3 + 2) = g(29) = \frac{1}{29}$

3. Solve the following equations for y.

a. $2xy + x^2 + sin(x) = 3y - 4xy + 2$

$$\leftrightarrow 2xy - 3y + 4xy = 2 - x^2 + \sin(x)$$

$$\leftrightarrow y(2x - 3 + 4x) = 2 - x^2 + \sin(x)$$

$$\rightarrow y = \frac{2 - x^2 + \sin(x)}{6x - 3}$$

b. $sin(y) + 5 = x^2 + 3x + 2$

$$\leftrightarrow \sin(y) = x^2 + 3x - 3$$

$$\rightarrow y = \sin^{-1}(x^2 + 3x - 3)$$

c. $\log_{17} y = x + 2$

$$\rightarrow y = 17^{x+2}$$

d. $\frac{1}{y+2} + x^2 = cos(x)$

$$\leftrightarrow 1 + x^2(y + 2) = \cos(x)\,(y + 2)$$

$$\leftrightarrow 1 + x^2y + 2x^2 = ycos(x) + 2\cos(x)$$

$$\leftrightarrow x^2y - ycos(x) = 2\cos(x) - 1 - 2x^2$$

$$\rightarrow y = \frac{2\cos(x) - 1 - 2x^2}{x^2 - \cos(x)}$$

e. $(x + 2y)^2 - 3x + sin(0) = 4y^2 + 2xy$

$$\leftrightarrow x^2 + 4xy + 4y^2 - 3x + 0 = 4y^2 + 2xy$$

$$\leftrightarrow x^2 + 2xy - 3x = 0$$

$$\rightarrow y = \frac{3x - x^2}{2x} = \frac{3 - x}{2}$$

Evaluate the following value if it exists.

a. $\sin\left(\frac{5\pi}{2}\right) = 1$

b. $\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$

c. $\sec(0) = 1$

d. $\cos\left(\frac{7\pi}{3}\right) = \frac{1}{2}$

e. $\tan\left(-\frac{\pi}{4}\right) = -1$

f. $\cot\left(\frac{13\pi}{6}\right) = \sqrt{3}$

g. $\csc(2\pi) \rightarrow Undefined$

5. Simplify the following expressions

a. $(a^2)^3 + (3+b)^2$

$$\rightarrow a^6 + 9 + 6b + b^2$$

b. $(x^4 + 7x^4 - 5x^2)^2$

$$\rightarrow (8x^4 - 5x^2)^2 = 64x^8 - 80x^6 + 25x^4$$

c. $4(5-3p) - (2+4p) - 3(2+10p)$

$$\rightarrow 20 - 12p - 2 - 4p - 6 - 30p = 12 - 46p$$

d. $\sin(0) + \ln(e) + \cos(\pi)$

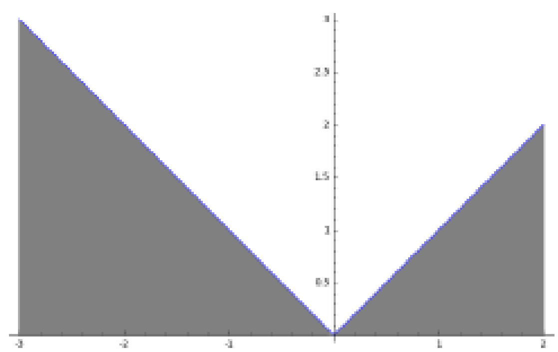
$$\rightarrow 0 + 1 + (-1) = 0$$

e. $\frac{t^{-3}+s^2t^2+5}{t^5}$

$$\rightarrow \frac{1}{t^8} + \frac{s^2}{t^3} + \frac{5}{t^5} = t^{-8} + s^2t^{-3} + 5t^{-5} = \frac{1 + s^2t^5 + 5t^3}{t^8}$$

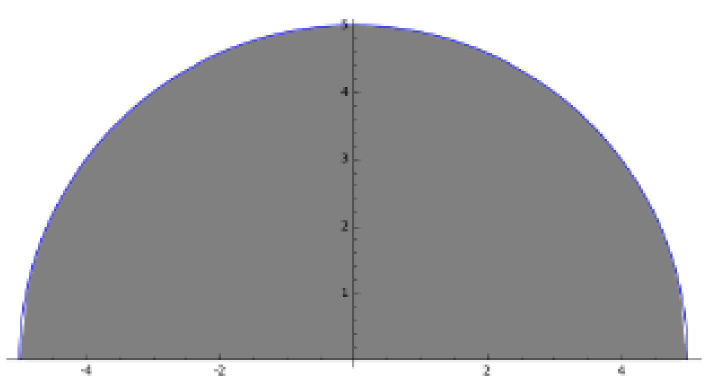
6. Find the area of the shaded regions below.

a. Area between $f(x) = |x|$ and the $x-$axis from $x = -3$ to $x = 2$.



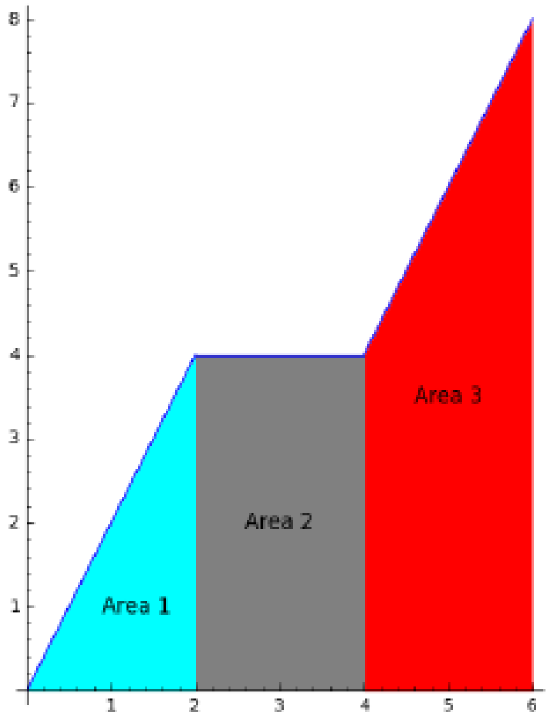
$$Area\ of\ 2\ Triangles: \frac{1}{2}(3)(3) + \frac{1}{2}(2)(2) = \frac{9}{2} + \frac{4}{2} = \frac{13}{2}$$

b. Area between $f(x) = \sqrt{25 - x^2}$ and the $x-$axis from $x = -5$ to $x = 5$.



$$Area\ of\ a\ Half-Circle: \frac{1}{2}\pi(5)^2 = \frac{25}{2}\pi$$

c. Area between the piecewise function below and the $x-axis$ from $x = 0$ to $x = 6$.



$$Area\ 1 \rightarrow \frac{1}{2}(2)(4) = 4$$

$$Area\ 2 \rightarrow (2)(4) = 8$$

$$Area\ 3 \rightarrow 2 \cdot 4 + \frac{1}{2}(2)(4) = 8 + 4 = 12\ or\ \frac{4+8}{2} \cdot 2 = 12$$

$$Total\ Area: 4 + 8 + 12 = 24$$

7. Find all vertical and horizontal asymptotes for the following functions

a. $\frac{1}{(x-3)^2}$

$$V.A.\ @\ x = 3$$

$$H.A.\ @\ y = 0$$

b. $\frac{2x-5}{(5x-3)}$

$$V.A.\ @\ x = \frac{3}{5}$$

$$H.A.\ @\ y = \frac{2}{5}$$

c. $\frac{(x-3)(x-2)(x+5)}{x+4}$

$$V.A.\ @\ x = -4$$

$$No\ H.A.$$

d. $\frac{(2x+3)(4x-7)}{x^2-4}$

$$V.A. @\ x = \pm 2$$

$$H.A. @\ y = 8$$

8. Find a linear function $f(x)$ such that $f(3) = 7$ and $f(-1) = \frac{3}{2}$.

$$\textit{Line that contains the points } (3,7)\ \&\ \left(-1, \frac{3}{2}\right) \rightarrow y = \frac{11}{8}x + \frac{23}{8}$$

9. Find the $x-$ intercepts for the following functions.

a. $f(x) = (x-3)^2(x^2+4x+3) = (x-3)(x-3)(x+3)(x+1)$

$$\rightarrow x - int: (3,0), (-3,0), (-1,0)$$

b. $g(\theta) = \sin(\theta)$

$$\textit{Infinitely Many } x - intercepts: (0,0), (\pm\pi, 0), (\pm 2\pi, 0), \dots$$

$$\textit{In general, there are } x - \textit{interecepts at the points } x = k\pi, \textit{where } k \textit{ is an integer } (k \in \mathbb{Z})$$

c. $h(x) = \frac{5}{x-3} + 1$

$$\textit{Set } h(x) = 0 \rightarrow \frac{5}{x-3} + 1 = 0 \rightarrow \frac{5}{x-3} = -1 \rightarrow 5 = 3 - x \rightarrow x = -2$$

$$\rightarrow x - int: (-2,0)$$

10. Find the distance between the points $P(2,5)$ and $Q(3,-7)$.

$$\sqrt{(2-3)^2 + (5+7)^2} = \sqrt{5^2 + 12^2} = \sqrt{25+144} = \sqrt{169} = 13$$

11. Rewrite each of the following radical expressions in exponential form, and then simplify the result as much as possible.

a. $\sqrt[3]{x}$

$$\sqrt[3]{x} = x^{1/3}$$

b. $\sqrt[3]{x^{12}}$

$$\sqrt[3]{x^{12}} = x^{12/3} = x^4$$

c. $\left(\sqrt[2]{x}\right)^5$

$$\left(\sqrt[2]{x}\right)^5 = x^{5/2}$$

d. $\sqrt[2]{x^5}$

$$\sqrt[2]{x^5} = x^{5/2}$$

e. $\frac{1}{\sqrt[3]{x^2}}$

$$\frac{1}{\sqrt[3]{x^2}} = \frac{1}{x^{2/3}} = x^{-2/3}$$

f. $\frac{\sqrt[2]{x}}{\sqrt[3]{x}}$

$$\frac{\sqrt[2]{x}}{\sqrt[3]{x}} = \frac{x^{1/2}}{x^{1/3}} = x^{\frac{1}{2}-\frac{1}{3}} = x^{1/6}$$

g. $\sqrt[3]{x}\left(\frac{1}{\sqrt{x}} + x - 2 + \frac{1}{\sqrt[3]{x}}\right)$

$$\sqrt[3]{x}\left(\frac{1}{\sqrt{x}} + x - 2 + \frac{1}{\sqrt[3]{x}}\right) = x^{1/3}\left(x^{-1/2} + x - 2 + x^{-1/3}\right) = x^{-1/6} + x^{4/3} - 2x^{1/3} + 1$$

h. $\frac{\sqrt[7]{x^2}+\sqrt[3]{x^3}}{\sqrt{x}}$

$$\frac{\sqrt[7]{x^2} + \sqrt[3]{x^3}}{\sqrt{x}} = \frac{x^{2/7} + x}{x^{1/2}} = x^{-1/2}\left(x^{2/7} + x\right) = x^{-3/14} + x^{1/2}$$