TAFTCOLLEGE

Math 2120 Analytic Geometry and Calculus II

Catalog Description:

A continuation of Mathematics 2100, this course includes integration; techniques of integration; infinite sequences and series; polar and parametric equations; applications of integration. Primarily for Science, Technology, Engineering, & Math Majors. C-ID: MATH 221. Transfer Credit: CSU; UC.

SLO:

Course #1 - Apply integration to physical problems.
Course #2 - Differentiate and Integrate functions of parametric equations and polar coordinates.
Course #3 - Apply an infinite series to a physical problem.

Sample Problems:

1. Rewrite the following radicals in exponential form.

$a)\ \sqrt[2]{x^3}$

$b)\ \left(\sqrt[4]{3x}\right)^3$

$c)\sqrt[4]{x^3y}$

$d)\ \dfrac{\sqrt[3]{x}}{\sqrt{x}}$

2. Graph the following exponential functions. Identify the domain and specify all vertical and horizontal asymptotes.

$a)\ y = \ 2^x$

$b)\ y = 2(2^x) + 1$

$c)\ y \ = \ -4(3^x) - 2$

3. Find the horizontal and vertical asymptotes of the following rational functions.

a) $f(x) = \frac{3x+2}{x^2-1}$

b) $f(x) = \frac{4}{x-2}$

c) $f(x) = \frac{3x^2+6x+5}{x^2-3x+2}$

4. Rewrite the following logarithms as exponents.

a) $9 = \log_x 5$

b) $y = \log_5 7$

c) $11 = \log_4 x$

5. Graph the following logarithmic functions. Identify the domain for each function.

a) $y = -\log_4(x)$

b) $y = \log_{10}(3x)$

c) $y = \log_2(x+3)$

6. Find the asymptotes of the logarithmic functions.

a) $y = \log_{10}(2x+5)$

b) $y = \log_2(x)$

c) $y = \log_{10}(x^2-9)$

7. Rewrite the following exponential functions as logarithms.

a) $5^{2t-3} = 5$

b)$10^4 = 10000$

c) $x = 10^{-2.2}$

8. Use partial fraction decomposition to rewrite the following fractions.

a) $\dfrac{6t+1}{t^2+t}$

b) $\dfrac{5x+3}{(x+1)(x+3)}$

c) $\dfrac{2x+1}{(x+5)^2}$

9. In reviewing limit rules, use the sum and difference rules to solve for the following limits.

a) $\lim\limits_{x \to 2}(5x^2 + 2x)$

b) $\lim\limits_{x \to 4}\big(f(x) - g(x)\big)$ where $f(x) = \frac{1}{2}\mathrm{x} - 7$ and $g(x) = 5x$

c) $\lim\limits_{x \to \infty}\left(4 + \dfrac{20}{x^2}\right)$

10. Use L'Hopital's Rule to evaluate the following limits.

a) $\lim\limits_{x \to 1}\left(\dfrac{x^3 + x^2 - 2x}{x - 1}\right)$

b) $\lim\limits_{x \to \infty}\left(\dfrac{3 - 4x^2 + 5x^3}{12x^3 + 10x^2 - 15x + 6}\right)$

c) $\lim\limits_{x \to 0}\left(\dfrac{\sin(x)}{x}\right)$

11. Use the power and sum rule to find the following derivatives..

a) $\frac{d}{dx}\left(x^3 + x^{-\frac{1}{3}} + \sqrt[4]{x} + 2\right)$

b) $\frac{d}{dx}\left(x^{-5} - x^{-\frac{2}{3}} + \sqrt[5]{x} + \pi\right)$

12. Use the Product Rule to find the following derivatives.

a) $\frac{d}{dx}[(x^2 + 3x)(5x + 2)]$

b) $\frac{d}{dx}[(5x^2)(\sin(x))]$

13. Use the quotient rule to find the following derivatives.

a) $\frac{d}{dx}\left(\frac{x+2}{3x+1}\right)$

b) $\frac{d}{dt}\left(\frac{(t+1)^3}{17}\right)$

14. Use the chain rule to find the following derivatives.

a) $\frac{d}{dx}\cos(x^5)$

b) $\frac{d}{dx}\cos^5(x)$

c) $\frac{d}{dx} f\left(h(g(3))\right)$ *where* $f(x) = 3x^2, g(x) = x, h(x) = 2x$

15. Find the derivative of the following function.

$$\frac{d}{dx}(\sin(x) + \cos(x) + \tan(x) + \cot(x) + \sec(x) + \csc(x))$$

16. Use the power rule to evaluate the following integrals.

a) $\int x^3 dx$

b) $\int 5x^{-2} dx$

c) $\int 3x^3 + 4x dx$

d) $\int \sqrt{s} + \sqrt[3]{s} + \sqrt[4]{s} ds$

17. Use $u-$ substitution to evaluate the following integrals.

a) $\int \cos^3(x)\sin(x)\, dx$

b) $\int x^3(x^4 + 6)^8 dx$

c) $\int 5x\sqrt{2x+3}\, dx$

d) $\int_{-1}^{2} x^2(x^3 + 2)^{-3} dx$

e) $\int_{-1}^{2} 6t(t^2 - 1)^2 dt$

18. Evaluate the following trigonometric integrals.

a) $\int \sin(r) + \cos(r)\, dr$

b) $\int \sec^2(3\theta) d\theta$

c) $\int \csc(4x) \cot(4x)\, dx$

d) $\int \sin^3(x)\, dx$

19. Evaluate the following sums.

a) $\sum_{k=1}^{4} \frac{5k+3}{k+2}$

b) $\sum_{k=1}^{6} k^3 + 2k$

c) $\sum_{i=0}^{4} f(i)$, where $f(x) = x^2$

20. Suppose the velocity in m/s of an object moving along a line is given by the function $v(x) = x^2$, where $0 \leq x \leq 8$. Approximate the displacement of the object by dividing the time interval $[0, 8]$ into n subintervals of equal length. On each subinterval, approximate the velocity by a constant equal to the value of v evaluate at the midpoint of the subinterval. Using $n = 8$ subintervals.

21. Factor and solve the following quadratic equations.

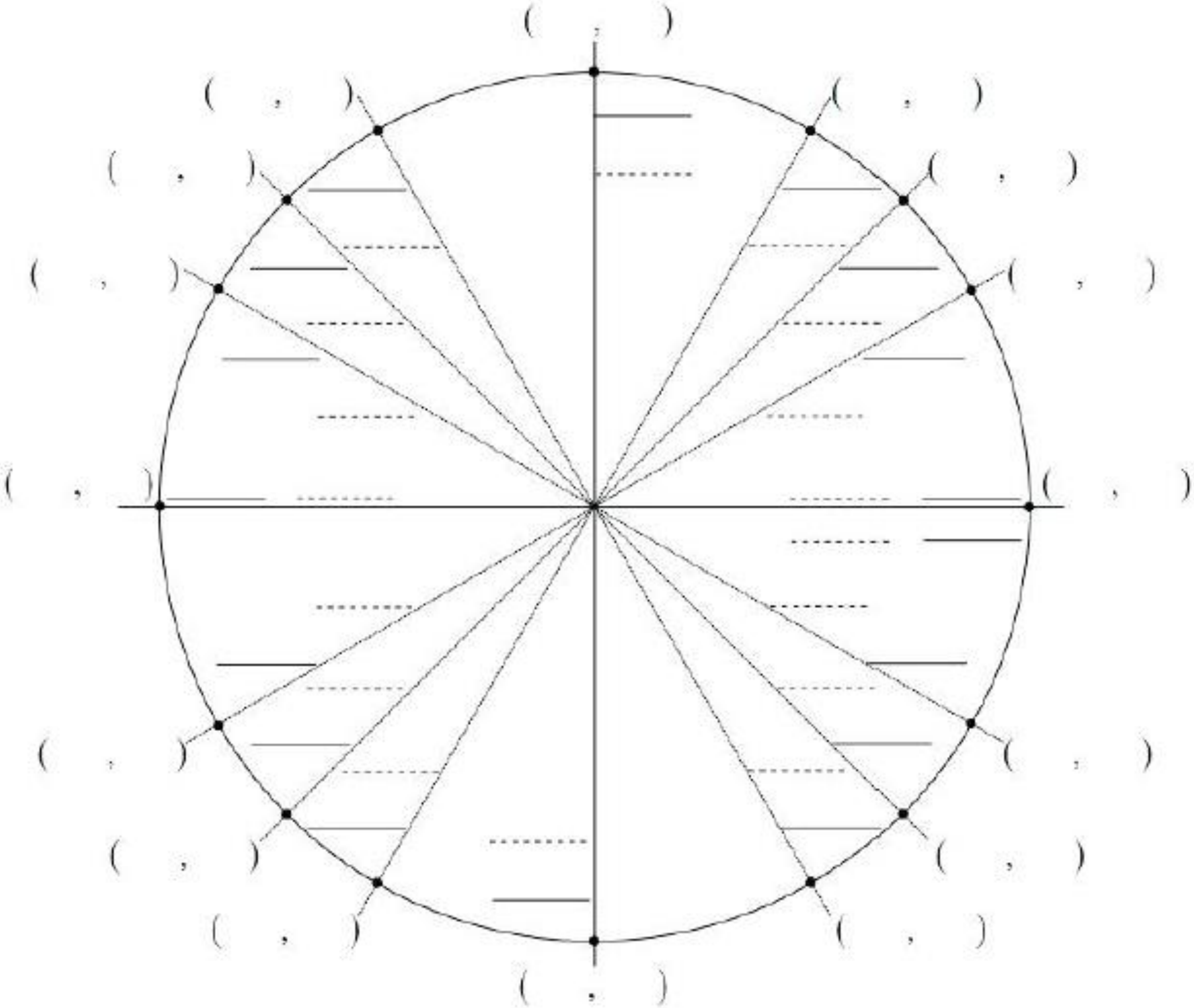
a. $x^2 - 2x + 1 = 0$

b. $2x^2 - x - 1 = 0$

c. $6x^2 + 10x = 14$

d. $2x^2 + 9x - 7 = 0$

22. Complete the unit circle. Include the angles in both degrees and in radians.



Solutions:

1. Rewrite the following radicals in exponential form.

a) $\sqrt[2]{x^3} \rightarrow x^{3/2}$

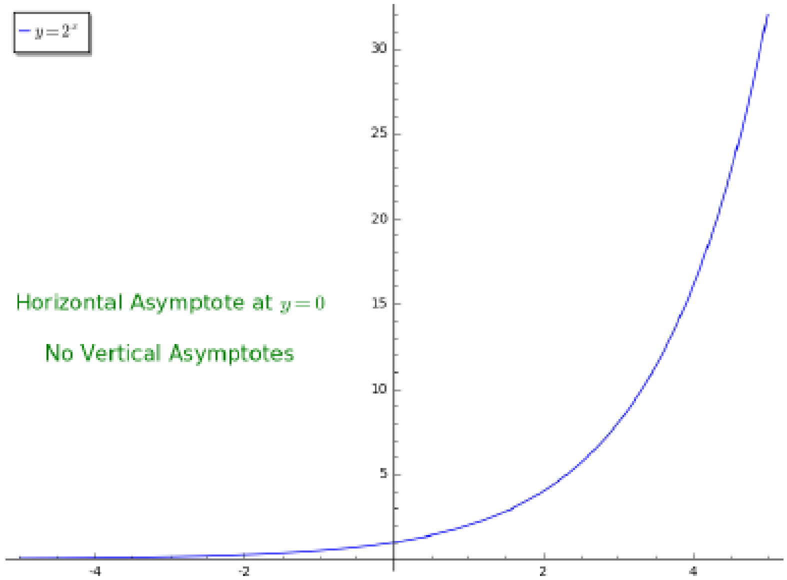
b) $\left(\sqrt[4]{3x}\right)^3 \rightarrow (3x)^{3/4}$

c) $\sqrt[4]{x^3y} \rightarrow (x^3y)^{1/4} = x^{3/4}y^{1/4}$

d) $\frac{\sqrt[3]{x}}{\sqrt{x}} \rightarrow \frac{x^{1/3}}{x^{1/2}} = x^{\frac{1}{3}-\frac{1}{2}} = x^{-1/6} = \frac{1}{x^{1/6}}$

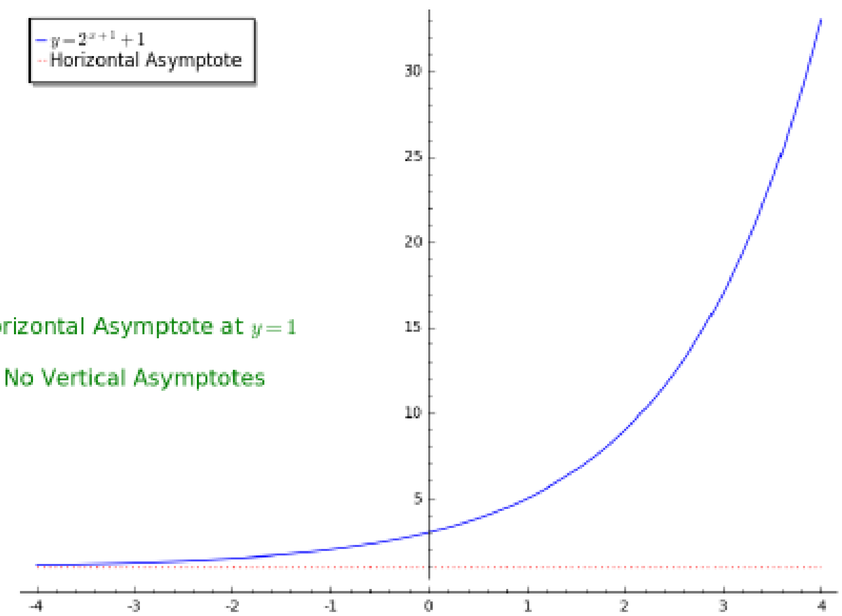
2. Graph the following exponential functions. Identify the domain and specify all vertical and horizontal asymptotes.

$a)\ y = 2^x$



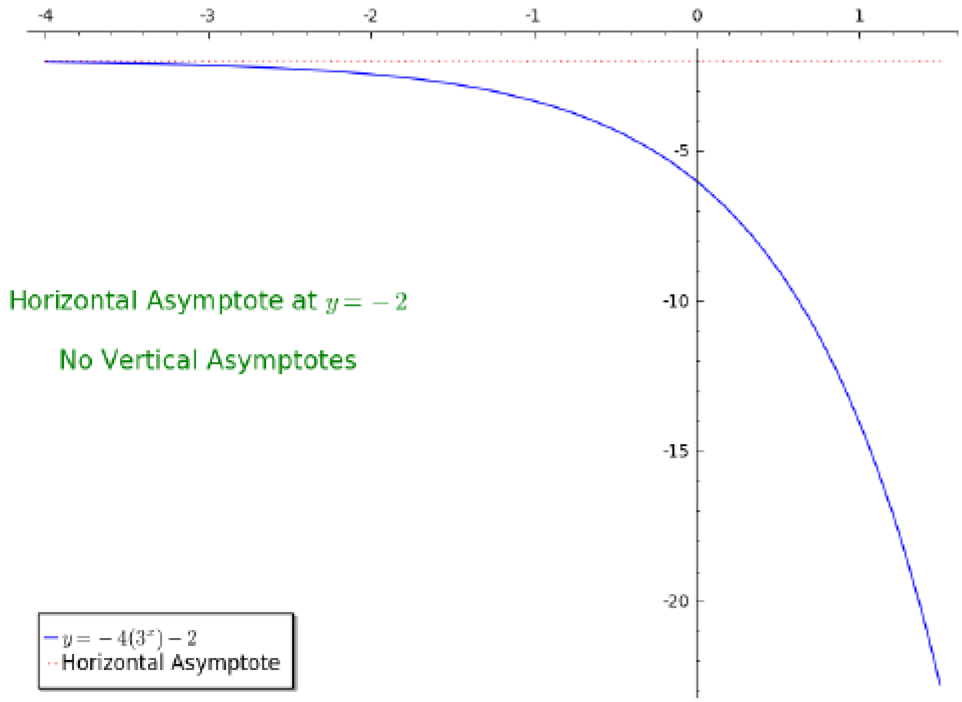
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$

$b)\ y = 2(2^x) + 1 \leftrightarrow y = 2^{x+1} + 1$



Domain: $(-\infty, \infty)$
Range: $(1, \infty)$

$c)\ y = -4(3^x) - 2$



Domain: $(-\infty, \infty)$
Range: $(-\infty, -2)$

3. Find the horizontal and vertical asymptotes of the following rational functions.

$a)\ f(x) = \frac{3x+2}{x^2-1}$

$$Horizontal: y = 0$$

$$Vertical: x = \pm 1$$

$b)\ f(x) = \frac{4}{x-2}$

$$Horizontal: y = 0$$

$$Vertical: x = 2$$

$c)\ f(x) = \frac{3x^2+6x+5}{x^2-3x+2} = \frac{3x^2+6x+5}{(x-1)(x-2)}$

$$Horizontal: y = 3$$

$$Vertical: x = 1\ \&\ x = 2$$

4. Rewrite the following logarithms as exponents.

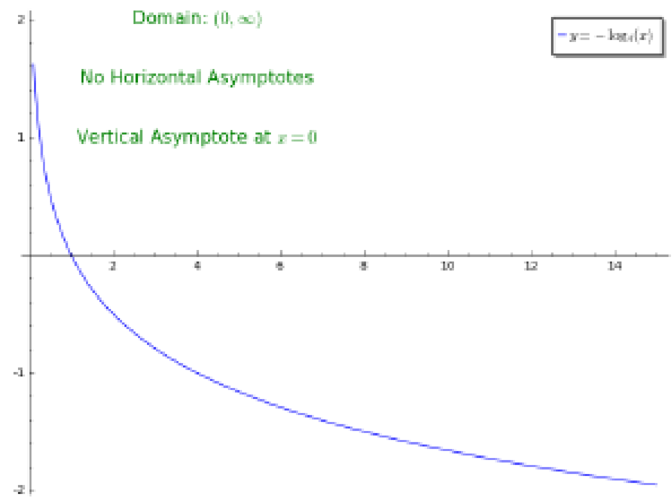
$a)\ 9 = \ log_x 5 \rightarrow x^9 = 5$

$b)\ y = \ log_5 7 \rightarrow 5^y = 7$

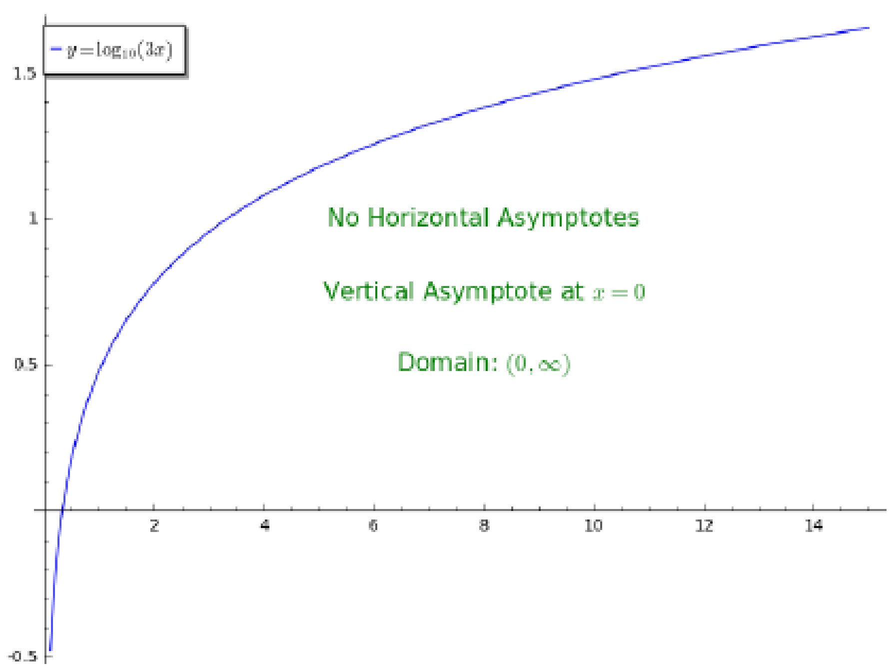
$c)\ 11 = log_4 x \rightarrow 4^{11} = x$

5. Graph the following logarithmic functions. Identify the domain for each function.

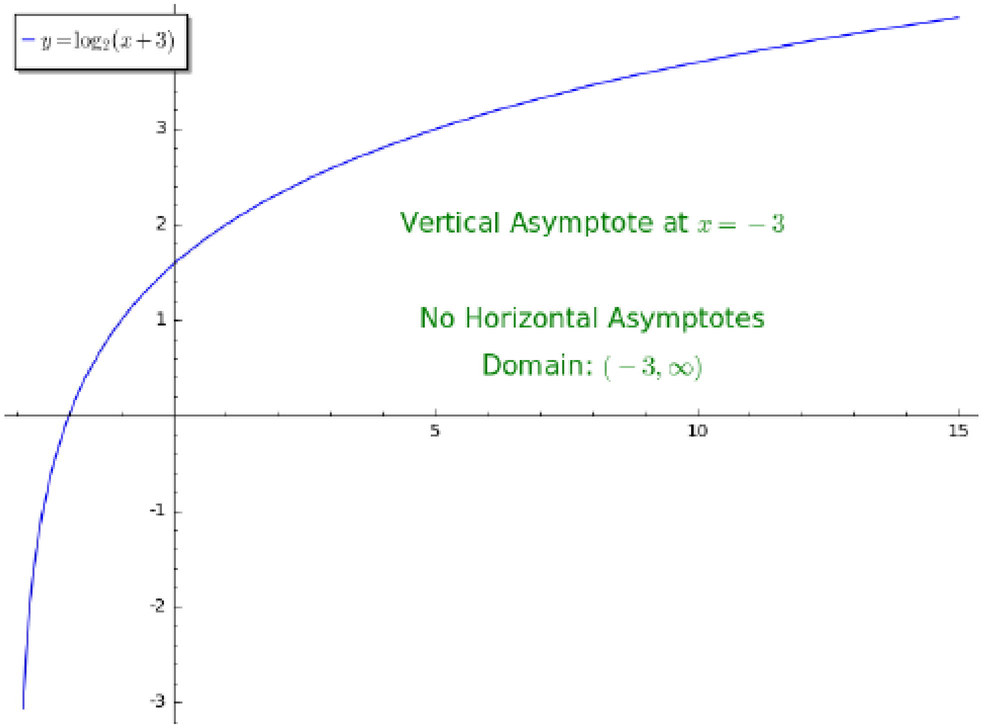
a) $y = -log_4(x)$



b) $y = \log_{10}(3x)$



c) $y = log_2(x+3)$



6. Find the asymptotes of the logarithmic functions.

a) $y = log_{10}(2x+5)$

$$2x + 5 = 0 \rightarrow x = -\frac{5}{2}$$

$$Horizontal: None$$

$$Vertical: x = -\frac{5}{2}$$

b) $y = log_2(x)$

$$Horizontal: None$$

$$Vertical: x = 0$$

c) $y = log_{10}(x^2 - 9)$

$$x^2 - 9 = 0 \rightarrow x = \pm 3$$

$$Horizontal: None$$

$$Vertical: x = \pm 3$$

7. Rewrite the following exponential functions as logarithms.

a) $5^{2t-3} = 5 \rightarrow \log_5(5) = 2t - 3 \leftrightarrow 1 = 2t - 3$

b) $10^4 = 10000 \rightarrow \log_{10}(10000) = 4$

c) $x = 10^{-2.2} \rightarrow \log_{10}(x) = -2.2$

8. Use partial fraction decomposition to rewrite the following fractions.

a) $\dfrac{6t+1}{t^2+t}$

$$\frac{6t+1}{t^2+t} = \frac{A}{t} + \frac{B}{t+1}$$

$$\rightarrow 6t + 1 = A(t+1) + Bt$$

$$\rightarrow 6t + 1 = (A+B)t + A$$

$$\rightarrow A = 1 \,\&\, B = 5$$

$$\rightarrow \frac{6t+1}{t^2+t} = \frac{1}{t} + \frac{5}{t+1}$$

b) $\dfrac{5x+3}{(x+1)(x+3)}$

$$\frac{5x+3}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3}$$

$$\rightarrow 5x + 3 = A(x+3) + B(x+1)$$

$$\rightarrow 5x + 3 = (A+B)x + (3A+B)$$

$$\rightarrow A = -1 \,\&\, B = 6$$

$$\rightarrow \frac{5x+3}{(x+1)(x+3)} = \frac{-1}{x+1} + \frac{6}{x+3}$$

c) $\frac{2x+1}{(x+5)^2}$

$$\frac{2x+1}{(x+5)^2} = \frac{A}{x+5} + \frac{B}{(x+5)^2}$$

$$\rightarrow 2x+1 = A(x+5) + B$$

$$\rightarrow 2x+1 = Ax + (5A+B)$$

$$\rightarrow A = 2 \ \& \ B = -9$$

$$\rightarrow \frac{2x+1}{(x+5)^2} = \frac{2}{x+5} - \frac{9}{(x+5)^2}$$

9. In reviewing limit rules, use the sum and difference rules to solve for the following limits.

a) $\lim_{x\to 2}(5x^2 + 2x)$

$$\rightarrow 5 \cdot \lim_{x\to 2}(x^2) + 2 \cdot \lim_{x\to 2}(x) = 5(2)^2 + 2(2) = 24$$

b) $\lim_{x\to 4}\big(f(x) - g(x)\big)$ where $f(x) = \frac{1}{2}\text{x} - 7$ and $g(x) = 5x$

$$\lim_{x\to 4}\big(f(x) - g(x)\big) = \lim_{x\to 4}\left(\left(\frac{1}{2}x - 7\right) - (5x)\right) = \lim_{x\to 4}\left(-\frac{9}{2}x - 7\right)$$

$$\rightarrow -\frac{9}{2} \cdot \lim_{x\to 4}(x) - \lim_{x\to 4}(7) = -\frac{9}{2}(4) - 7 = -25$$

c) $\lim_{x\to\infty}\left(4 + \frac{20}{x^2}\right)$

$$\lim_{x\to\infty}\left(4 + \frac{20}{x^2}\right) = \lim_{x\to\infty}(4) + 20 \cdot \lim_{x\to\infty}\left(\frac{1}{x^2}\right) = 4 + 0 = 4$$

10. Use L'Hopital's Rule to evaluate the following limits.

a) $\lim\limits_{x\to 1}\left(\frac{x^3+x^2-2x}{x-1}\right)$

$$\lim_{x\to 1}\left(\frac{x^3+x^2-2x}{x-1}\right)=\frac{0}{0}$$

$$\to \lim_{x\to 1}\left(\frac{3x^2+2x-2}{1}\right)=\frac{3+2-2}{1}=3$$

b) $\lim\limits_{x\to\infty}\left(\frac{3-4x^2+5x^3}{12x^3+10x^2-15x+6}\right)$

$$\lim_{x\to\infty}\left(\frac{3-4x^2+5x^3}{12x^3+10x^2-15x+6}\right)=\frac{\infty}{\infty}$$

$$\to \lim_{x\to\infty}\left(\frac{-8x+15x^2}{36x^2+20x-15}\right)=\frac{\infty}{\infty}$$

$$\to \lim_{x\to\infty}\left(\frac{-8+30x}{72x+20}\right)=\frac{\infty}{\infty}$$

$$\to \lim_{x\to\infty}\left(\frac{30}{72}\right)=\frac{30}{72}=\frac{5}{12}$$

c) $\lim\limits_{x\to 0}\left(\frac{\sin(x)}{x}\right)$

$$\lim_{x\to 0}\left(\frac{\sin(x)}{x}\right)=\frac{0}{0}$$

$$\to \lim_{x\to 0}\left(\frac{\cos(x)}{1}\right)=\frac{\cos(0)}{1}=1$$

11. Use the power and sum rule to find the following derivatives.

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}(f(x)) + \frac{d}{dx}(g(x)) = f'(x) + g'(x)$$

$$\frac{d}{dx}(x^p) = px^{p-1}$$

a) $\frac{d}{dx}\left(x^3 + x^{-\frac{1}{3}} + \sqrt[4]{x} + 2\right)$

$$\rightarrow \frac{d}{dx}\left(x^3 + x^{-\frac{1}{3}} + x^{\frac{1}{4}} + 2\right) = 3x^2 - \frac{1}{3}x^{-\frac{4}{3}} + \frac{1}{4}x^{-\frac{3}{4}}$$

b) $\frac{d}{dx}\left(x^{-5} - x^{-\frac{2}{3}} + \sqrt[5]{x} + \pi\right)$

$$\frac{d}{dx}\left(x^{-5} - x^{-\frac{2}{3}} + x^{\frac{1}{5}} + \pi\right) = -5x^{-6} + \frac{2}{3}x^{-\frac{5}{3}} + \frac{1}{5}x^{-\frac{4}{5}}$$

12. Use the Product Rule to find the following derivatives.

$$\frac{d}{dx}[f(x) \cdot g(x)] = f'(x)g(x) + f(x)g'(x)$$

a) $\frac{d}{dx}[(x^2 + 3x)(5x + 2)]$

$$(2x + 3)(5x + 2) + (x^2 + 3x)(5)$$

$$\rightarrow 15x^2 + 34x + 6$$

b) $\frac{d}{dx}[(5x^2)(\sin(x))]$

$$(10x)(\sin(x)) + (5x^2)(\cos(x))$$

$$\rightarrow 5x[2\sin(x) + x \cdot \cos(x)]$$

13. Use the quotient rule to find the following derivatives.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}, \text{provided } g(x) \neq 0$$

a) $\frac{d}{dx}\left(\frac{x+2}{3x+1}\right)$

$$\frac{d}{dx}\left(\frac{x+2}{3x+1}\right) = \frac{(1)(3x+1) - (x+2)(3)}{(3x+1)^2} = \frac{-5}{9x^2+6x+3}$$

b) $\frac{d}{dt}\left(\frac{(t+1)^3}{17}\right)$

$$\frac{d}{dt}\left(\frac{(t+1)^3}{17}\right) = \frac{3(t+1)^2(17) - (t+1)^3(0)}{17^2} = \frac{51(t+1)^2}{289} = \frac{3(t+1)^2}{17}$$

14. Use the chain rule to find the following derivatives.

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

a) $\frac{d}{dx}\cos(x^5)$

$$\frac{d}{dx}\cos(x^5) = -\sin(x^5) \cdot 5x^4$$

b) $\frac{d}{dx}\cos^5(x)$

$$\frac{d}{dx}\cos^5(x) = 5\cos^4(x)\,(-\sin(x)) = -5x \cdot cos^4(x)\sin(x)$$

c) $\frac{d}{dx} f\left(h(g(3))\right)$ $where\ f(x) = 3x^2, g(x) = x, h(x) = 2x$

$$g(3) = 3$$

$$h(g(3)) = h(3) = 6$$

$$f\left(h(g(3))\right) = f(6) = 108$$

$$\frac{d}{dx} f\left(h(g(3))\right) = \frac{d}{dx}(108) = 0$$

15. Find the derivative of the following function.

$$\frac{d}{dx}(\sin(x)+\cos(x)+\tan(x)+\cot(x)+\sec(x)+\csc(x))$$

$$\rightarrow \frac{d}{dx}\sin(x)+\frac{d}{dx}\cos(x)+\frac{d}{dx}\tan(x)+\frac{d}{dx}\cot(x)+\frac{d}{dx}\sec(x)+\frac{d}{dx}\csc(x)$$

$$=\cos(x)-\sin(x)+\sec^2(x)-\csc^2(x)+\sec(x)\tan(x)-\csc(x)\cot(x)$$

16. Use the power rule to evaluate the following integrals.

$$\int x^p\,dx=\frac{x^{p+1}}{p+1}+C, \qquad p\neq -1$$

a) $\int x^3dx=\frac{x^4}{4}+C$

b) $\int 5x^{-2}dx=5\left(\frac{x^{-1}}{-1}\right)+C=-\frac{5}{x}+C$

c) $\int 3x^3+4xdx=3\left(\frac{x^4}{4}\right)+4\left(\frac{x^2}{2}\right)+C=\frac{3}{4}x^4+2x^2+C$

d) $\int \sqrt{s}+\sqrt[3]{s}+\sqrt[4]{s}ds=\frac{2}{3}s^{3/2}+\frac{3}{4}s^{4/3}+\frac{4}{5}s^{5/4}+C$

17. Use $u-$ substitution to evaluate the following integrals.

a) $\int \cos^3(x)\sin(x)\,dx \rightarrow$ *Let* $u=\cos(x)$ *then* $du=-\sin(x)dx \leftrightarrow -du=\sin(x)dx$

$$\leftrightarrow \int -u^3du=-\frac{u^4}{4}+C$$

$$(\textit{Substitute back}) \leftrightarrow -\frac{\cos^4(x)}{4}+C$$

$$\textit{Verify:}\frac{d}{dx}\left(-\frac{\cos^4(x)}{4}+C\right)=\cos^3(x)\sin(x)$$

b) $\int x^3(x^4+6)^8 dx \rightarrow Let\ u = x^4 + 6\ then\ du = 4x^3dx \leftrightarrow \frac{1}{4}du = x^3dx$

$$\leftrightarrow \int \frac{1}{4}u^8 du = \frac{1}{4}\cdot\frac{u^9}{9} + C = \frac{1}{36}u^9 + C$$

$$(Substitute\ back) \leftrightarrow \frac{1}{36}(x^4+6)^9 + C$$

$$Verify: \frac{d}{dx}\left(\frac{1}{36}(x^4+6)^9 + C\right) = x^3(x^4+6)^8$$

c) $\int 5x\sqrt{2x+3}\,dx \rightarrow Let\ u = 2x+3\ then\ du = 2dx \rightarrow \frac{1}{2}du = dx\ \&\ x = \frac{(u-3)}{2}$

$$\leftrightarrow \int 5\frac{(u-3)}{2}u^{1/2}\left(\frac{1}{2}\right)du = \frac{5}{4}\int u^{3/2} - 3u^{1/2}du = \frac{5}{4}\left(\frac{2}{5}u^{5/2} - 3\left(\frac{2}{3}\right)u^{3/2}\right) + C$$

$$(Substitute\ back\ and\ simplify) \leftrightarrow \frac{1}{2}(2x+3)^{5/2} - \frac{5}{2}(2x+3)^{\frac{3}{2}} + C$$

$$Verify: \frac{d}{dx}\left(\frac{1}{2}(2x+3)^{5/2} - \frac{5}{2}(2x+3)^{\frac{3}{2}} + C\right) = 5x\sqrt{2x+3}$$

d) $\int_{-1}^{2} x^2(x^3+2)^{-3}dx \rightarrow Let\ u = x^3 + 2\ then\ du = 3x^2dx \leftrightarrow \frac{1}{3}du = x^2dx$

$$\leftrightarrow \int_{LB}^{UB} \frac{1}{3}u^{-3}du = -\frac{1}{6}u^{-2}\Big|_{u=LB}^{u=UB}$$

$$(Substitute\ back) \leftrightarrow -\frac{1}{6}(x^3+2)^{-2}\Big|_{x=-1}^{x=2} \rightarrow -\frac{1}{6}[10^{-2} - (1)^{-2}] = \frac{99}{600} = \frac{33}{200}$$

Note: Instead of having to substitute back before evaluating our definite integral at the lower and upper bounds, we could solve for our bounds in terms of u using the substitution that we have already established.

$$Since\ u = x^3 + 2 \rightarrow UB = (2)^3 + 2 = 10 \quad \& \quad LB = (-1)^3 + 2 = 1$$

Then we end up with the following definite integral in terms of u:

$$\int_{1}^{10} \frac{1}{3}u^{-3}du = -\frac{1}{6}u^{-2}\Big|_{u=1}^{u=10} \rightarrow -\frac{1}{6}[10^{-2} - 1^{-2}] = -\frac{1}{6}\left[\frac{1}{100} - 1\right] = \frac{33}{200}$$

e) $\int_{-1}^{2} 6t(t^2-1)^2 dt \rightarrow Let\ u = (t^2-1)\ then\ du = 2dt \rightarrow 3du = 6dt$

$$\leftrightarrow \int_{LB}^{UB} 3u^2 du = u^3|_{u=LB}^{u=UB}$$

$$(Substitute\ back) \leftrightarrow (t^2-1)^3|_{t=-1}^{t=2} \rightarrow (3)^3 - (0)^3 = 27$$

Note: Instead of having to substitute back before evaluating our definite integral at the lower and upper bounds, we could solve for our bounds in terms of u using the substitution that we have already established.

$$Since\ u = t^2 - 1 \rightarrow UB = (2)^2 - 1 = 3 \quad \& \quad LB = (-1)^2 - 1 = 0$$

Then we end up with the following definite integral in terms of u:

$$\int_{0}^{3} 3u^2 du = u^3|_{u=0}^{u=3} \rightarrow (3)^3 - (0)^3 = 27$$

18. Evaluate the following trigonometric integrals.

a) $\int \sin(r) + \cos(r)\, dr = -\cos(r) + \sin(r) + C$

b) $\int \sec^2(3\theta) d\theta = \frac{1}{3}\tan(3\theta) + C$

c) $\int \csc(4x)\cot(4x)\, dx = -\frac{1}{4}\csc(4x) + C$

d) $\int \sin^3(x)\, dx = \int \sin(x)\sin^2(x)\, dx = \int \sin(x)\,[1-\cos^2(x)]\, dx$

$$= \int \sin(x)\, dx - \int \sin(x)\cos^2(x)\, dx$$

$$= -\cos(x) + \frac{1}{3}\cos^3(x) + C$$

19. Evaluate the following sums.

a) $\sum_{k=1}^{4} \frac{5k+3}{k+2} = \left[\frac{5(1)+3}{1+2}\right] + \left[\frac{5(2)+3}{2+2}\right] + \left[\frac{5(3)+3}{3+2}\right] + \left[\frac{5(4)+3}{4+2}\right] = \frac{8}{3} + \frac{13}{4} + \frac{18}{5} + \frac{23}{6} = \frac{267}{20}$

b) $\sum_{k=1}^{6} k^3 + 2k = [1^3 + 2(1)] + [2^3 + 2(2)] + [3^3 + 2(3)] + [4^3 + 2(4)] + [5^3 + 2(5)] + [6^3 + 2(6)]$

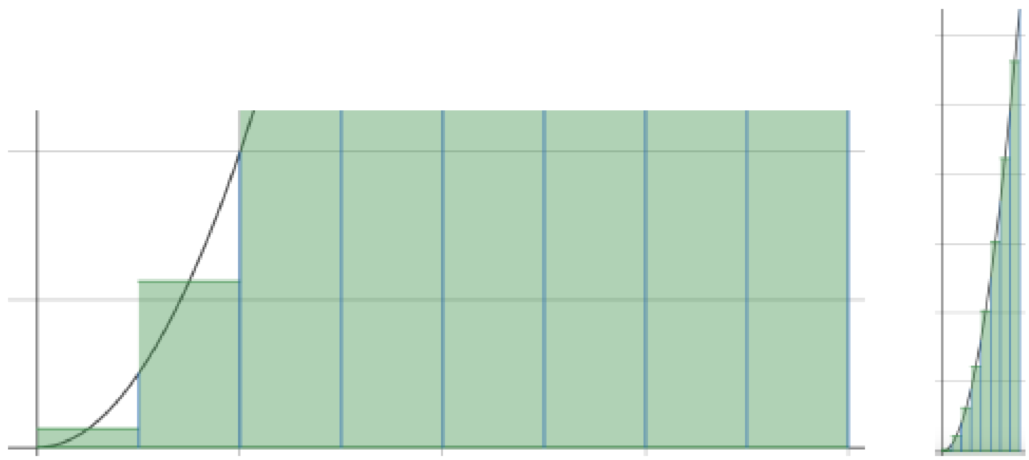
$$3 + 12 + 33 + 72 + 135 + 228 = 483$$

c) $\sum_{i=0}^{4} f(i)$, where $f(x) = x^2 \rightarrow f(0) + f(1) + f(2) + f(3) + f(4)$

$$\rightarrow 0^2 + 1^2 + 2^2 + 3^2 + 4^2 = 30$$

20. Suppose the velocity in m/s of an object moving along a line is given by the function $v(x) = x^2$, where $0 \leq x \leq 8$. Approximate the displacement of the object by dividing the time interval $[0, 8]$ into n subintervals of equal length. On each subinterval, approximate the velocity by a constant equal to the value of v evaluate at the midpoint of the subinterval. Using $n = 8$ subintervals.

(Riemann Sum Problem) The graph on the left below shows the rectangles on the first 2 intervals that are constructed using the midpoint method. The graph on the right shows all 8 rectangles for the midpoint method.



Width of Rectangles $\Delta x = \frac{8-0}{8} = 1$

Interval	Midpoint	Height at Midpoint	Area of Rectangle
[0,1]	0.5	$(0.5)^2 = 0.25$	$1(0.25) = 0.25$
[1,2]	1.5	$(1.5)^2 = 2.25$	$1(2.25) = 2.25$
[2,3]	2.5	$(2.5)^2 = 6.25$	$1(6.25) = 6.25$
[3,4]	3.5	$(3.5)^2 = 12.25$	$1(12.25) = 12.25$
[4,5]	4.5	$(4.5)^2 = 20.25$	$1(20.25) = 20.25$
[5,6]	5.5	$(5.5)^2 = 30.25$	$1(30.25) = 30.25$
[6,7]	6.5	$(6.5)^2 = 42.25$	$1(42.25) = 42.25$
[7,8]	7.5	$(7.5)^2 = 56.25$	$1(56.25) = 56.25$

Total Area = Sum of Rectangle Areas = 170

21. Factor and solve the following quadratic equations.

a. $x^2 - 2x + 1 = 0$

$$\rightarrow (x-1)^2 = 0 \rightarrow x = 1 \text{ with multiplicity} = 2$$

b. $2x^2 - x - 1 = 0$

$$\rightarrow (2x+1)(x-1) = 0$$

$$x = -\frac{1}{2} \;\&\; x = 1$$

c. $6x^2 + 10x = 14$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(6)(-14)}}{2(6)}$$

$$\rightarrow x = \frac{-10 \pm \sqrt{100 + 336}}{12} = \frac{-10 \pm \sqrt{436}}{12} = \frac{-10 \pm 2\sqrt{109}}{12} = \frac{-5 \pm \sqrt{109}}{6}$$

$$x = \frac{-5 - \sqrt{109}}{6} \;\&\; \frac{-5 + \sqrt{109}}{6}$$

d. $2x^2 + 9x - 7 = 0$

$$x = \frac{-9 \pm \sqrt{9^2 - 4(2)(-7)}}{2(2)}$$

$$\rightarrow x = \frac{-9 \pm \sqrt{81 + 56}}{4} = \frac{-9 \pm \sqrt{137}}{4}$$

$$x = \frac{-9 - \sqrt{137}}{4} \;\&\; x = \frac{-9 + \sqrt{137}}{4}$$

22. Complete the unit circle. Include the angles in both degrees and in radians.