TAFTCOLLEGE

Math 2130 Analytic Geometry and Calculus III

Catalog Description:

This is a continuation of Mathematics 2120 including vector functions and analysis, partial differentiation, vector valued functions, calculus of functions of more than one variable, partial derivatives, multiple integration, Green's Theorem, Stokes' Theorem, divergence theorem, multiple integration and line integrals. C-ID: MATH 230. Transfer Credit: CSU; UC.

SLO:

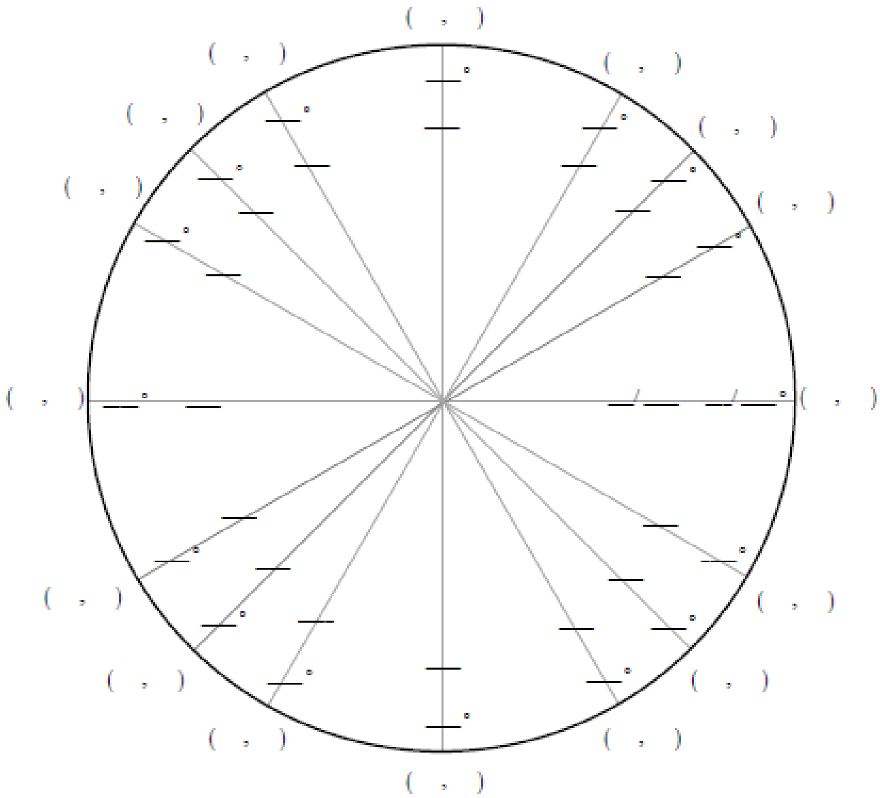
Course #1 - Apply vector calculus to physical problems.
Course #2 - State and apply basic definitions, properties and theorems of multivariable Calculus

Sample Problems:

Review for Calculus III

Instructions: Solve each of the following problems below. Make sure to keep track of any formulas or theorems that you use as your work through each problem. Also keep note of which problems you have questions on and bring those to class on the first day.

1. Complete the Unit Circle. Include the angles in both degrees and radians.



2. Calculate the following limits.
   a. $\lim_{x\to 1} \frac{x^3-7x^2+12x}{4-x}$
   b. $\lim_{x\to 3} \frac{x^4-81}{x-3}$
   c. $\lim_{x\to \frac{\pi}{4}} \frac{sin^2\theta - cos^2\theta}{sin\theta - cos\theta}$
3. Evaluate the following derivatives using the Product, Quotient or Chain Rules.
   a. $\frac{d}{dt}(5t^2 \, sin \, t)$
   b. $\frac{d}{du}\left(\frac{4u^2+u}{8u+1}\right)$
   c. $\frac{d}{dx}(5x + sin^3 \, x + sin \, x^3)$
   d. $\frac{d}{dv}\left(\frac{v}{3v^2+2v+1}\right)^{\frac{1}{3}}$
   e. $\frac{d}{dx}(2x(\sin x)\sqrt{3x-1})$
   f. $\frac{d}{dx}\left(e^{x^2} + \ln(x) + \sin(x)\right)$
   g. $\frac{d}{d\theta}(\sin(cos(2\theta)))$

4. Implicit Differentiation Calculate y'(x) for $y = \frac{cos \, y}{1+sin \, x}$.
5. Find the critical points of $2x^3 - 3x^2 - 36x + 12$ on the interval $(-\infty, \infty)$. Identify the absolute maximum and minimum values.
6. Find the Critical points of $f(x) = x^2(x^2 + 4x - 8); [-5,2]$ on the given interval and determine the absolute extreme values of f(x) on the given interval.
7. Use L'Hospitals Rule to evaluate the following limits:
   a. $\lim_{t\to 2} \frac{t^3-t^2-2t}{t^2-4}$
   b. $\lim_{x\to \infty} \frac{5x^2+2x-5}{\sqrt{x^4-1}}$
8. Determine the following indefinite integrals.
   a. $\int (x^8 - 3x^3 + 1)dx$
   b. $\int (\sin 2\theta + 2\theta + 1)d\theta$
   c. $\int \left(\frac{1}{x^2} - \frac{2}{x^{5/2}}\right) dx$
   d. $\int (4x^{1/3} - 7x^{2/5} + 10x^{3/7})dx$
9. Use a change of variables (u substitution) to find the following indefinite integrals.
   a. $\int \frac{x}{(x-2)^3} dx$
   b. $\int x\cos x^2 dx$
   c. $\int \frac{x}{\sqrt{x-4}} dx$
   d. $\int \frac{\sin x}{\cos^2 x} dx$
10. Region between curves.
    a. Find the region bounded by $y = \sqrt{x}, y = 2x - 1$and $y = 0$.
    b. Find the region bounded by $y = \sqrt{x-1}, y = 2, y = 0$ and $x = 0$.
11. Evaluate and simplify the following derivatives.
    a. $\frac{d}{dx}(xe^{-10x})$
    b. $\frac{d}{dx}(2^{x^2-x})$
    c. $\frac{d}{dx}(\log_3(x+8))$
12. Evaluate the following integrals using Integration by parts
    a. $\int x^2 e^{4x} dx$
    b. $\int x^2 \sin 2x dx$

c. $\int e^{3x}\cos 2x dx$

13. Evaluate the following integrals using Trigonometric Substitution

a. $\int_0^{3/2} \frac{dx}{(9-x^2)^{3/2}}$

b. $\int \frac{dx}{\sqrt{x^2-49}}, x > 7$

c. $\int \frac{dx}{\sqrt{16+4x^2}}$

14. Evaluate the following integrals using Partial Fractions.

a. $\int \frac{3}{x^2-1} dx$

b. $\int \frac{3}{x^3-9x^2} dx$

c. $\int \frac{z+1}{z(z^2+4)} dz$

15. Evaluate the following integrals or state that they diverge.

a. $\int_1^\infty x^{-2} dx$

b. $\int_2^\infty \frac{dx}{\sqrt{x}}$

c. $\int_0^1 \frac{x^3}{x^4-1} dx$

16. Evaluate the following integrals:

a. $\int \sec^2(x) + 2 + \sin(4) + x^3 dx$

b. $\int 4x^2 + 5x^{-3} + \sin\left(\frac{x}{2}\right) dx$

c. $\int \sqrt{s} + \sqrt[3]{s} + \sqrt[4]{s} ds$

d. $\int \sin^4(t)\cos^3(\mathrm{t})\,\mathrm{dt}$

e. $\int_{-1}^{2} 6t(t^2-1)^2 dt$

f. $\int x^2 e^x dx$

g. $\int \tan(t) + \ln(t)\, dt$

h. $\int (x^2y^3 + \sin(xy))dy$

i. $\int \frac{3x+11}{x^2-x-6} dx$

17. Evaluate the following limits or explain why they do not exist.

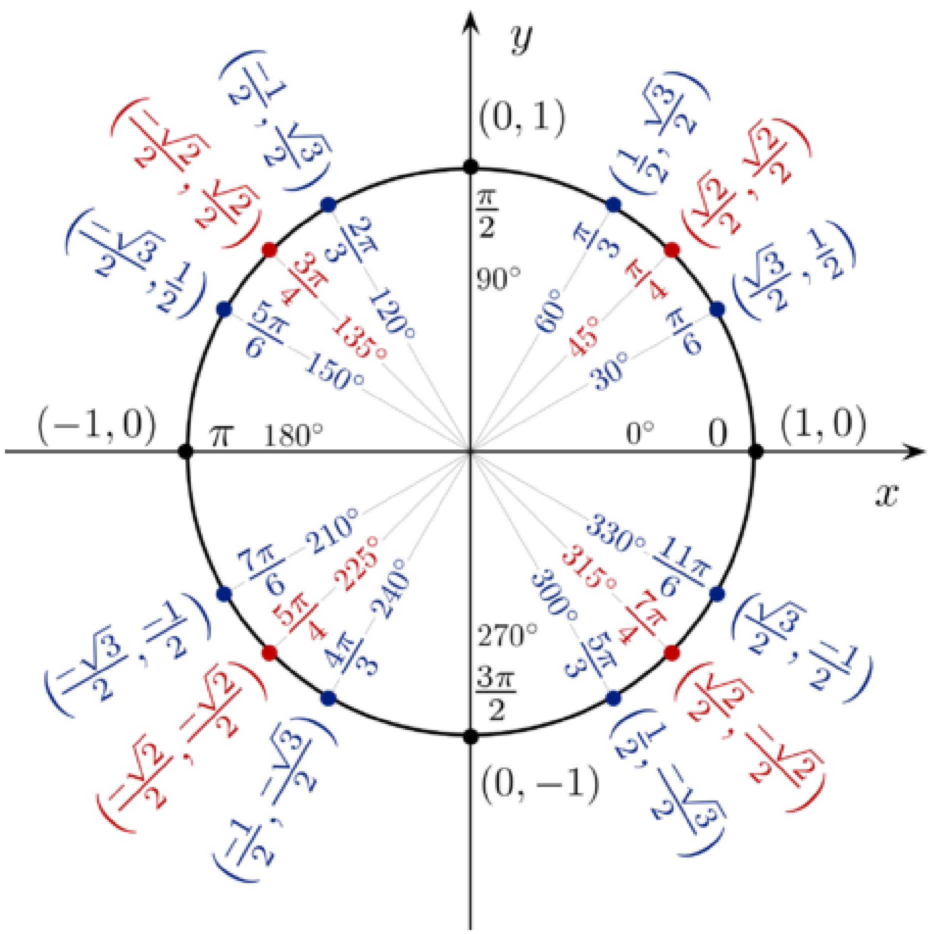
a. $\lim_{\mathrm{x}\to 0}(1+4\mathrm{x})^{3\mathrm{x}}$

b. $\lim_{\mathrm{x}\to\infty} \frac{\ln \mathrm{x}^{100}}{\sqrt{\mathrm{x}}}$

18. Find the volume of the solid that is generated when the region bounded by $f(x) = e^{-x}$ and the x-axis on $[1, e^2]$ is revolved around the x-axis.

Solutions

1. Complete the Unit Circle. Include the angles in both degrees and radians.



2. Calculate the following limits.
   a. $\lim_{x\to 1} \frac{x^3-7x^2+12x}{4-x} = 2$
   b. $\lim_{x\to 3} \frac{x^4-81}{x-3} = 108$
   c. $\lim_{x\to \frac{\pi}{4}} \frac{sin^2\theta-cos^2\theta}{sin\theta-cos\theta} = \sqrt{2}$
3. Evaluate the following derivatives using the Product, Quotient or Chain Rules.
   g. $\frac{d}{dt}(5t^2\, sin\, t) = 5t^2 \cos t + 10t \sin t$
   h. $\frac{d}{du}\left(\frac{4u^2+u}{8u+1}\right) = \frac{32u^2+8u+1}{(8u+1)^2}$
   i. $\frac{d}{dx}(5x + sin^3\, x + sin\, x^3) = 5 + 3\sin^2 x \cos x + 3x^2\cos(x^3)$
   j. $\frac{\mathrm{d}}{\mathrm{dv}}\left(\frac{\mathrm{v}}{3v^2+2v+1}\right)^{1/3} = \frac{1-3v^2}{3v^{3/2}\sqrt{3v^2+2v+1}}$

$$\frac{d}{dx}(2x(\sin x)\sqrt{3x-1}) =$$

k. $2(\sin x)\sqrt{3x-1} + 2x(\cos x)\sqrt{3x-1} + \frac{3x\sin x}{\sqrt{3x-1}} =$

$\frac{9x\sin x - 2\sin x + 6x^2\cos x - 2x\cos x}{\sqrt{3x-1}}$

l. $\frac{d}{dx}\left(e^{x^2} + \ln(x) + \sin(x)\right) = 2xe^{x^2} + \frac{1}{x} + \cos x$

g. $\frac{d}{d\theta}(\sin(cos(2\theta))) = -2\sin(2\theta)\cos(\cos(2\theta))$

4. Implicit Differentiation Calculate y'(x) for $y = \frac{cos\ y}{1+sin\ x} = \frac{-y\cos x}{1+\sin x+\sin y}$.
5. Find the critical points of $2x^3 - 3x^2 - 36x + 12$ on the interval $(-\infty, \infty)$. Identify the absolute maximum and minimum values. Critical Points $x = -3\ and\ x = -2$. No absolute max or min.
6. Find the Critical points of $f(x) = x^2(x^2 + 4x - 8); [-5,2]$ on the given interval and determine the absolute extreme values of f(x) on the given interval. Critical points $x = -4, x = 0, x = 1$. Abs max of 16 at $x = 2$. Abs min of -128 at $x = -4$.
7. Use L'Hospitals Rule to evaluate the following limits:
   c. $\lim_{t\to 2} \frac{t^3-t^2-2t}{t^2-4} = \frac{3}{2}$
   d. $\lim_{x\to\infty} \frac{5x^2+2x-5}{\sqrt{x^4-1}} = 5$
8. Determine the following indefinite integrals.
   e. $\int (x^8 - 3x^3 + 1)dx = \frac{x^9}{9} - 3\cdot\frac{x^4}{4} + x + C = \frac{x^9}{9} - \frac{3}{4}x^4 + x + C$
   f. $\int (\sin 2\theta + 2\theta + 1)d\theta = -\frac{1}{2}\cos 2\theta + \theta^2 + \theta + C$
   g. $\int (\frac{1}{x^2} - \frac{2}{x^{5/2}})dx = -x^{-1} - 2\cdot(-\frac{2}{3})x^{-3/2} = -\frac{1}{x} + \frac{4}{3}x^{-3/2} + C$
   h. $\int (4x^{1/3} - 7x^{2/5} + 10x^{3/7})dx = 3x^{4/3} - 5x^{7/5} + 7x^{10/7}$
9. Use a change of variables (u substitution) to find the following indefinite integrals.
   a. $\int \frac{x}{(x-2)^3}dx = -\frac{x-1}{(x-2)^2} + C$
   e. $\int x\cos x^2 dx = \frac{1}{2}\sin x^2 + C$
   f. $\int \frac{x}{\sqrt{x-4}}dx = \frac{2}{3}\cdot(x-4)^{3/2} + 8\sqrt{x-4} + C$
   g. $\int_0^{\pi/4} \frac{\sin x}{\cos^2 x}dx = \frac{2}{\sqrt{2}} - 1 = \sqrt{2} - 1$
10. Area of the region between curves.
    c. Find the area of the region bounded by $y = \sqrt{x}, y = 2x - 1$and $y = 0.\ Area = \frac{63}{4}$
    d. Find the area of the region bounded by $y = \sqrt{x-1}, y = 2, y = 0$ and and $x = 0$.
    e. $Area = \frac{14}{3}$
11. Evaluate and simplify the following derivatives.
    d. $\frac{d}{dx}(xe^{-10x}) = e^{-10x} + x(-10e^{-10x}) = e^{-10x}(1 - 10x)$
    e. $\frac{d}{dx}(2^{x^2-x}) = 2^{x^2-x}\cdot\ln 2\cdot(2x-1)$
    f. $\frac{d}{dx}(\log_3(x+8)) = \frac{1}{(x+8)\ln 3}$
12. Evaluate the following integrals using Integration by parts
    d. $\int x^2e^{4x}dx = \frac{1}{4}x^2e^{4x} - \frac{1}{8}xe^{4x} + \frac{1}{32}e^{4x} + C$
    e. $\int x^2\sin 2x dx = \frac{-1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + C$
    f. $\int e^{3x}\cos 2x dx = \frac{3}{13}e^{3x}(\cos 2x + \frac{2}{3}\sin 2x) + C = \frac{1}{13}e^{3x}(3\cos 2x + 2\sin 2x) + C$
13. Evaluate the following integrals using Trigonometric Substitution

d. $\int_0^{3/2} \frac{dx}{(9-x^2)^{3/2}} = \frac{\sqrt{3}}{27}$

e. $\int \frac{dx}{\sqrt{x^2-49}}, x > 7 = \ln|\frac{x}{7} + \frac{\sqrt{x^2-49}}{7} + c$. Note that the abs value signs can be omitted since $x > 7$, and if we replace $-\ln(7) + C$by a different arbitrary constant D, we can write the result as $\ln(x + \sqrt{x^2 - 49} + D$.

f. $\int \frac{dx}{\sqrt{16+4x^2}} = \frac{1}{2}\ln|\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}| + C$

14. Evaluate the following integrals using Partial Fractions.

d. $\int \frac{3}{x^2-1} dx = \int (\frac{3/2}{x-1} - \frac{3/2}{x+1})dx = \frac{3}{2}(\ln|x-1| - \ln|x+1|) + C$

e. $\int \frac{3}{x^3-9x^2} dx = \int \left(\frac{-\frac{1}{27}}{x} - \frac{\frac{1}{3}}{x^2} + \frac{\frac{1}{27}}{x-9}\right) dx = \frac{1}{27}(\ln|x-9| - \ln|x|) + \frac{1}{3x} + C$

f. $\int \frac{z+1}{z(z^2+4)} dz = \int \left(\frac{1}{4z} - \frac{z}{4(z^2+4)} + \frac{1}{z^2+4}\right) dz = \frac{1}{4}\ln|z| - \frac{1}{8}\ln(z^2+4) + \frac{1}{2}\tan^{-1}\left(\frac{z}{2}\right) + C$

15. Evaluate the following integrals or state that they diverge.

d. $\int_1^\infty x^{-2} dx = 1$

e. $\int_2^\infty \frac{dx}{\sqrt{x}}$ Diverges

f. $\int_0^1 \frac{x^3}{x^4-1} dx$ Diverges

16. Evaluate the following integrals:

j. $\int \sec^2(x) + 2 + \sin(4) + x^3 dx = \frac{x^4}{4} + 2x + x\sin x + \frac{\sin(2x)}{\cos(2x)+1} + c$

k. $\int 4x^2 + 5x^{-3} + \sin\left(\frac{x}{2}\right) dx = \frac{4}{3}x^3 + \frac{5}{2x^2} - 2\cos\frac{x}{2} + C$

l. $\int \sqrt{s} + \sqrt[3]{s} + \sqrt[4]{s} ds = \frac{2}{3}s^{3/2} + \frac{3}{4}s^{4/3} + \frac{4}{5}s^{5/4} + C$

m. $\int \sin^4(t)\cos^3(\mathrm{t})\,\mathrm{dt} = \frac{1}{70}\sin^5 t(5\cos(2t) + 9) + C$

n. $\int_{-1}^2 6t(t^2-1)^2 dt = 27$

o. $\int x^2 e^x dx = e^x(x^2 = 2x + 2) + C$

p. $\int \tan(t) + \ln(t)\,dt = -t + t\,ln(t) - \ln(\cos t)$

q. $\int (x^2y^3 + \sin(xy))dy = \frac{x^2y^4}{4} - \frac{\cos(xy)}{x} + C$

r. $\int \frac{3x+11}{x^2-x-6} dx = 4\ln(3-x) - \ln(x+2) + C$

17. Evaluate the following limits or explain why they do not exist.

c. $\lim_{\mathrm{x}\to 0}(1 + 4\mathrm{x})^{3\mathrm{x}} = e^{12}$

d. $\lim_{\mathrm{x}\to\infty} \frac{\ln \mathrm{x}^{100}}{\sqrt{\mathrm{x}}} = 0$

18. Find the volume of the solid that is generated when the region bounded by $f(x) = e^{-x}$ and the x-axis on $[1, e^2]$ is revolved around the x-axis. Volume $= \frac{2\pi}{27}(13e^6 - 1)$.